Transaction costs and real firms: a suggested formalisation

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Introduction

In his Nobel Prize acceptance speech Oliver Williamson (2009) outlines the nature of his transaction cost (TC) project and suggests areas in need of progression. One such area involves formalisation of theory. He claims (p471)

> Transaction cost economics is sometimes criticized because it has not been fully formalized, to which I have three responses: transaction costs economics, like other theories, has undergone a natural progression; full formalization is a work-in-progress; and premature formalization runs the risk of a disconnection with the phenomena.

The current paper can be understood as part of this work-in-progress. The key objectives are threefold:

1. Integrate TCs into a standard model of the firm.
2. Examine the interaction between organisational factors (TCs) and standard demand-cost factors.
3. Analyse key propositions of transaction cost economics (TCE) with the general model.

This paper is, of course, not the only attempt at formalising TCE and organisational aspects of the firm: Grossman and Hart (1986), Hart and Moore (1990), Tadelis (2002), Bajari and Tadelis (2001) are examples. But a specific characteristic of the current discussion is an emphasis on "real" firms, as highlighted in the title given above. This builds on the TC tradition of avoiding "disconnection with the phenomena", to use Williamson's formulation in the above quotation. This emphasis on real firms builds on the formulation suggested by Coase (1937, 1993) that we need a "realistic" theory of the firm rather than the "blackboard economics" (Coase 1991) that characterises much writing. The somewhat standard view, as argued by Demsetz (1983, p377), is that we should not "confuse the firm of economic theory with its real-

world namesake. The chief mission of neoclassical economics is to understand how the price system coordinates the use of resources, not the inner workings of real firms". But this creates a rather artificial barrier between the "theory of the firm" (or the technical firm) and the "economics of the firm" (or the institutional firm). The result is that the analysis of real firms is viewed as an aspect of institutional analysis, and hence part of the economics of the firm, because technical firms are viewed only as an aspect of price theory.

For current purposes we view real firms as being both institutional and technical (see Dietrich and Krafft 2011, 2012). The institutional firm characteristically concerns itself with issues such as internal structure, organisation and boundaries. The theory of the firm analyses behaviour in particular market contexts in terms of demand-cost interaction. Both perspectives on the firm are relevant for analysis because the market context and demand-cost interaction can interact with institutional factors such as organisation and boundaries. An important driver of the current discussion is that these interconnections are (potentially) important, hence objective two of the three objectives for this paper highlighted above. It will be shown below that modelling interconnections between institutional and technical aspects of the firm qualifies some accepted conclusions. But because of the potential complexity of the interactions a formal method is useful.

The rest of the discussion is organised as follows. In the next section a number of general issues involved with modelling real firms are set out and in particular the formal analysis of transaction costs. Following this a trivially simple model of a single firm with no TCs is presented. The objective here is to identify the additional impact when TCs are introduced. The analytical framework is then augmented by the introduction of TCs. Following derivation of key results, and to facilitate further analysis, particular parameterisations are used based on a constant elasticity unit production cost function. This formulation allows particular conclusions to be drawn that can both reproduce standard TC logic but also deviate from this logic depending on parameter values. In the penultimate section firm viability issues are considered in terms of a non-negative profitability constraint. Introducing viability qualifies but does not fundamentally change earlier analysis. Finally conclusions are drawn.

Modelling real firms: general issues

The technical analysis presented here will be based on a single firm model apart from more general discussion and concluding comments. All relationships are assumed continuous and (apart from demand functions and organisational costs) twice differentiable. We assume linear demand; there is no real loss of generality here. TCs are viewed in the standard way as the costs of search, negotiation and policing that accompany any transaction i.e. exchange of a good or service. This exchange need not be market based but can also be exchange within a firm. To render the analysis tractable we build on the early TC tradition (Coase 1937, Williamson 1975) that views the governance choice as between markets and intra-firm hierarchies rather than the complexity of institutional forms involving "intermediate" governance structures analysed by more recent work (for example Williamson, 1985).

The extent of TCs is modelled in terms of organisational effort. To gain more information, and use this information during negotiation or policing, any firm requires additional organisational factor inputs. We therefore ignore dynamic and developmental issues and instead rely on a comparative static framework as does TCE. We therefore ignore issues, such as those suggested by Penrose (1959), in which firm growth and evolution can be based on managerial excess capacity. In this dynamic context acquisition and use of information need not be based on additional organisational factor inputs because of managerial excess capacity. The "organisational technology" defines the relationship between effort and TCs. This technology is summarised in terms of the first derivative of the relationship (see below). Using standard TC theory the link between effort and TCs will depend on the fundamental characteristics of any transaction, perhaps most notably (using Williamson, 1985) uncertainty and asset specificity. With high levels of the latter characteristics the first derivative of the TC relationship will be large. For example, an increase in organisational inputs used for search activity will involve a greater increase in TCs (with constant input prices per unit) if the search activity is complex because of uncertainty and/or asset specificity factors.

The linkages between organisational factors and standard demand-cost factors are modelled here in terms of firm revenues and average production costs being endogenous to organisational effort. For example, greater search or negotiation can

result in a more effective pricing policy i.e. we can expect a non-negative relationship between output price and effort (with the assumed single product firm). For average production costs we assume a non-positive relationship with effort. The reasoning here is that, for example, extra effort might result in more effective management of production, and hence greater technical efficiency and lower average costs. Alternatively extra effort might control post-contractual opportunism more effectively in input markets. In abstract terms we can think of a notional production cost frontier that is asymptotically approached with extra effort.

Allowing for both production activity and organisational factors implies that we have two measures of firm size: physical activity (measured for example as real output) and transaction costs, with the latter assumed to have a monotonically increasing relationship with its organisational "input" i.e. effort. This implies that the standard institutional analysis of the "existence of firms" can be recast here as a requirement for positive organisational effort. Hence the existence of firms (positive effort and real output), in the modelling presented below, is both an organisational and technical feature.

Obviously any real firm undertakes many specific organisational tasks that cover the management of output markets, intra-firm activity and input markets. To render the modelling tractable we assume that these various managerial tasks are undertaken in fixed proportions with, in addition, no substitution between organisational human and non-human inputs being possible. This allows us to analyse a "basket" of organisational tasks in which the internal characteristics of the basket are fixed although the whole basket may change size in a continuous manner. This simplification allows us to create an aggregate measure of transaction costs that is simply the sum of the various specific organisational costs (that are not modelled). We therefore have a single measure of organisational activity that can be applied in different contexts.

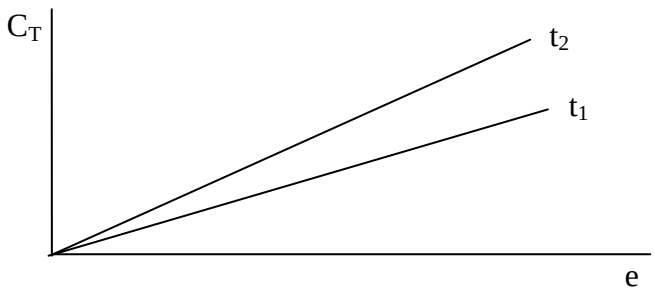
To model organisational activity we view our aggregate measure of transaction costs (C_T) as a function of organisation effort (e), an index of organisation input prices (p_O) and an index of transaction specific factors (t). In general terms:

$$C_T = f(e, p_O, t) \qquad (1)$$

Effort can be measured as organisational input per unit time. In principle this is unbounded from above. In addition both C_T (the organisational "output") and e (the "input") provide measures of organisation size. Throughout the discussion we assume p_O constant. The index t can be conceptualised as a ranking of possible exogenous features that determine transaction complexity. Greater complexity implies, ceteris paribus, greater transaction costs with constant e and p_O i.e. there is a direct effect of t on C_T. In addition, increasing complexity will increase the costs associated with search, negotiation and policing activity for any given change in effort i.e. there is also an indirect effect of t on C_T. Using standard TC reasoning this complexity will depend on (most notably) asset specificity and transaction uncertainty.

A corollary of the "organisational basket" assumption made above is that organisational technology can be viewed as linear along the lines suggested in Figure 1 below. With constant p_O, transaction costs are depicted for two levels of transaction complexity $t_2 > t_1$. Given this conceptualisation, it follows that increasing transaction complexity will increase f'(e). This reasoning is used below where f'(e) is used as an exogenous determinant of equilibrium effort. In addition increasing f'(e) is viewed as reflecting transaction complexity and furthermore can be used an indicator of underlying transaction characteristics.

Figure 1



We can link this general discussion of the modelling presented here to more conventional TC theory. Within a simple markets-hierarchies framework, traditional TCE uses a characteristic comparative institutional approach. This can be presented in the following way. For the production of a particular good or service indentify all

necessary transactions or organisational tasks. Note here that the characteristics of the real good or service are independent of the institutional analysis. In Williamson's (1985, p 22) words: "Holding the nature of the good or service to be delivered constant, economizing takes place..." Each transaction can be undertaken either within the firm using hierarchies, for which there is a transaction cost c_H, or external to the firm using markets, for which there is a transaction cost c_M. For each transaction the difference $c_M - c_H$ can be computed. Using a standard formulation, adopted here, the size of any difference depends on transaction complexity. Starting from the largest difference, an efficient firm will then internalise transactions until $c_M - c_H = 0$. If then a change in contracting conditions takes place, i.e. transaction complexity changes, that effects all transactions equivalently, an efficient firm will change its boundaries, involving more or less market activity, until $c_M - c_H = 0$ is once again attained. If the change in transaction complexity affects the various transactions differently, the ranking of the difference $c_M - c_H$ changes but the end result is the same: an increase or reduction in internalisation. In terms of the formulation used here, a change in transaction complexity affects organisational effort. We can note that there can be some low level contracting complexity where internal firm organisation is not rational i.e. $c_M - c_H$ is non-positive for all transactions, even if this low level of complexity exists only theoretically rather than in practice.

The framework developed here has the following obvious differences to a comparative institutional logic. First we assume a basket of organisational tasks that can vary in size but not internal structure. This facilitates the technical analysis based on profit maximisation with two choice variables: physical output and organisational effort. An implication is that this "organisational basket" approach avoids any requirement for a comparative institutional method. But we are still able to draw the key conclusion that with minimal contracting complexity internal firm organisation is not rational. We will see below that the framework developed here has this feature. At small f'(e) there is no positive equilibrium effort i.e. internal organisation is not rational. But above some threshold f'(e) a viable organisational equilibrium exists. In addition, increasing f'(e) results in greater equilibrium effort, consistent with greater internalisation , as would be predicted by traditional TCE. With plausible parameter values consistent with standard TC theory this is the only possible solution i.e. the framework can reproduce results consistent with existing TCE. But with different

parameter values, that are also plausible, a second class of equilibria can be derived that does not suggest increasing organisational effort with increasing f'(e). Hence this formal analysis of real firms suggests a partial but important departure from standard TC theory.

A single firm model with no transaction costs

Initially we set out a trivially simple model of a single firm with no TCs. This is a base model that allows identification of the specific impacts of organisational factors to be introduced below. We use a standard linear inverse demand function

$$p(x) = \alpha - \beta x \qquad (2)$$

p = selling price; x = output and α and β are positive constants. We express total production costs as:

$$C(x, a(x)) = xa(x) \qquad (3)$$

where a(x) is average production cost. We express total production costs in this way as later we allow average production costs to be endogenous to organisational effort and hence to TCs. Profit is then:

$$\pi(x, a(x)) = \alpha x - \beta x^2 - xa(x) \qquad (4)$$

The first order condition for maximum profit is:

$$\frac{\partial \pi}{\partial x} = \alpha - 2\beta x - a(x) - xa'(x) = 0 \qquad (5)$$

Here a'(x) indicates the nature of production based scale economies which become important later.

Rearranging the first order condition:

$$x = (\alpha - a(x))/(2\beta + a'(x)) \qquad (5a)$$

This involves the standard positive profit viability condition for positive output: $\alpha > a(x)$. This standard result is the key finding of this section. The second order condition is presented in the appendix.

A single firm model with transaction costs

This section develops the model just presented by introducing organisational effort (e). The augmented linear inverse demand function is given by:

$$p(x, e) = \alpha - \beta x + \delta e \qquad (6)$$

Where α, β and δ are positive constants. The logic here is that stated above: greater search, negotiation and/or policing activity can increase selling price without a reduction in demand. Or with a simple reformulation of the function: increasing organisational effort can increase demand without a lower selling price.

We define average production costs $(\bar{a})$ that are now endogenous to organisational effort. Total costs are the sum of production and organisation costs:

$$\bar{C} = x\bar{a}(x,e) + f(e) \qquad (7)$$

The nature of this cost function is further discussed below. The profit function is now given by:

$$\pi(x,\bar{a}(x,e),e) = \alpha x - \beta x^2 + \delta xe - x\bar{a}(x,e) - f(e) \qquad (8)$$

The firm has two choice variables: output and effort. The first order conditions for maximum profit are given by:

$$\frac{\partial \pi}{\partial x} = \alpha - 2\beta x + \delta e - \bar{a}(x,e) - x\bar{a}_x(x,e) = 0 \qquad (9a)$$

$$\frac{\partial \pi}{\partial e} = \delta x - x\bar{a}_e(x,e) - f'(e) = 0 \qquad (9b)$$

Based on earlier discussion we can offer the following interpretation of the partial derivatives:

$\bar{a}_e(x,e) \leq 0$ defines the organisational effort impact on unit production costs.

$\bar{a}_x(x,e) <> 0$ is an indicator of production scale effects: constant/ increasing/ decreasing returns to scale imply this derivative is zero/negative/positive.

f'(e) ≥ 0 is an indicator of transaction complexity.

Using (9a) we can define a condition for equilibrium real output:

$$x = \frac{\alpha - \bar{a}(x,e) + \delta e}{2\beta + \bar{a}_x(x,e)} \qquad (10a)$$

We see in (10a) a development of the earlier stated (non-organisational) condition for viable positive real output. The earlier condition (5a) is a special case here with e=0. The formulation in (10a) shows that profit maximising output is not unique but is a positive function of organisational effort. It is straightforward to show that we might

expect the effort-output relationship in (10a) to have a shape defined by $\partial x/\partial e > 0$ and $\partial^2 x/\partial e^2 < 0$. Initially assume constant returns to scale in production at all output levels i.e. $\bar{a}_x = 0$. Using (10a), define:

$$\frac{\partial x}{\partial e} = \frac{\delta}{2\beta} - \frac{\bar{a}_e}{2\beta}$$

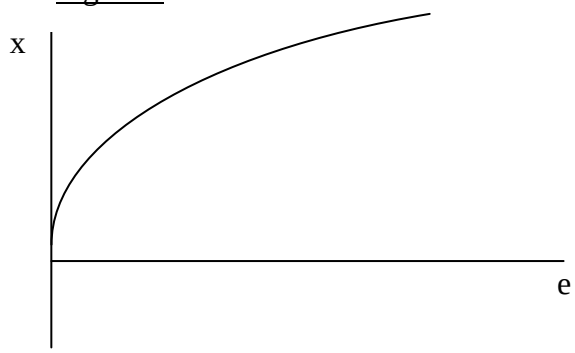
By assumption $\bar{a}_e$ is negative hence $\partial x/\partial e$ is positive for all parameter values. Defining the second derivative:

$$\frac{\partial^2 x}{\partial e^2} = -\frac{\bar{a}_{ee}}{2\beta}$$

Subject to $\bar{a}_{ee}$ being positive the second derivative is negative. Below a particular formulation for average production costs is used in which the impact of effort is defined by a constant elasticity. This formulation has the characteristic that $\bar{a}_{ee}$ is positive.

This general depiction of (10a) is shown in Figure 2. Later discussion shows that a constraint on maximum e is produced by a non-negative profit viability condition. In addition this constraint is produced by transaction complexity. So the analytical logic implies that complexity determines organisational effort and in turn effort determines physical output.

Figure 2



Although this discussion is based on the case of $\bar{a}_x = 0$ the results are more general. Consider the case of increasing returns to scale in production but the $\bar{a}_x < 0$ being

constant. Subject to 2β> $\bar{a}_x$ (which is a requirement for a relevant second order condition for a maximum: see the appendix) this case shifts the curves in the diagram up. For the case of diminishing returns in production the curves in the diagram will shift down. In addition we can point out that any positive level of effort produces positive profit maximising output, subject to the second order requirement of α greater than average production costs. Finally we can point out with respect to Figure 2 the desirable property that an increase in overall market size (α) will shift up the curve but not change the slope defined by the derivatives.

This discussion indicates that any solution for the level of physical output requires a solution to organisational characteristics (i.e. e). A more determinate result can be found by solving for equilibrium effort. Using (9b):

$$x = \frac{f'(e)}{\delta - \bar{a}_e(x,e)} \qquad (10b)$$

Combining (10a) and (10b) we can define equilibrium organisational effort in terms of the fundamentals of the model:

$$e = f'(e)\frac{\left[(2\beta + \bar{a}_x(x,e)\right]}{\delta^2 - \delta\bar{a}_e(x,e)} - \frac{\left[\alpha - \bar{a}(x,e)\right]}{\delta} \qquad (11)$$

The condition defined in (11) shows a relationship between f'(e), interpreted as contract complexity, and organisational effort, with the nature of the relationship being determined by the fundamentals of the model. The second order conditions for a maximum profit (see the appendix) suggest that the second term on the right hand side of (11) defines a threshold impact of f'(e) on effort. At low f'(e) no positive organisational activity is possible - a feature of (11) that appears consistent with standard TC theory as discussed above. But the "slope" impact of f'(e) on effort can be positive or negative.

Further analysis of (11) will restrict itself to the case of constant returns in production at all output levels i.e. $\bar{a}_x = 0$. This allows the discussion to concentrate on the interaction of production and organisational matters rather than production directly. In addition discussion will consider the interaction of two factors: potential market size (α) and the extent to which production costs respond to effort $(\bar{a}_e)$. If we adopt a market power interpretation of α, the marrying of these two factors (competition and

the effect of effort on costs) is logical. Output market dominance can only be maintained if there are defences against new firm entry. New firms will have entry opportunities if production cost advantages can be developed based on organisational activity (here summarised as effort). When average production costs are responsive to effort, new firm entry, and hence competitive markets, are likely. In conventional terms, entry in such conditions will be based on absolute cost advantages from organisational effort. With firm entry our single firm model is more accurately one of monopolistic competition rather than monopoly with entry reducing the size of α. We will see that equilibrium conditions for e depend on these two factors: α and $(\bar{a}_e)$.

As average production costs and its first derivative with respect to effort are themselves functions of effort, it is useful to use the assumption of constant returns and re-write (11) in the following form:

$$\frac{[\delta - \bar{a}_e(x,e)][\alpha - \bar{a}(x,e) + \delta e]}{2\beta} = f'(e) \qquad (11a)$$

It is clear from (11a) that the relationship between contract complexity (i.e. f'(e)) and organisational activity is a complex function of effort. Because of this complexity, further discussion of equilibrium effort will be based around a particular formulation. For all cases a constant elasticity unit production cost function with constant returns is used:

$$\bar{a} = x^0 e^{\varepsilon} \qquad (12)$$

In (12) ε is non-positive. Using (12) the formulation in (11a) can be re-written:

$$\frac{[\delta - \varepsilon e^{(\varepsilon-1)}][\alpha - e^{\varepsilon} + \delta e]}{2\beta} = f'(e) \qquad (11b).$$

The constant elasticity formulation in (12), or (11b), suggests that there is no "control loss" as organisational size (e) increases i.e. ε is unchanged at different levels of effort. This feature is somewhat different from organisational discussion that suggests incentive, motivation and similar issues as organisational size increases. As a robustness check on this characteristic of (12) the results reported below were reproduced with a trans-log formulation:

$$\bar{a} = x^0 e^{(\varepsilon 1 + \varepsilon 2 e)} \qquad (12a)$$

In this latter formulation $\varepsilon 1$ is non-positive and $\varepsilon 2$ is strictly positive. Subject to the condition that the overall exponent on effort is negative, the results are structurally the same as those reported for (12) and hence are not reported here. In addition the discussion of firm viability, in the next section, is relevant here. This suggests that at some large level of effort profitability becomes negative and hence places a constraint on organisational size. So, while there is no direct organisational control loss in (12) there is a similar indirect effect that results from the interaction of technical and organisational factors. It is perhaps more significant that organisational control loss can be derived in this was rather than imposed as a characteristic of a particular functional form.

The formulation reported in (11b) is examined below. In these results two values for ε are used. Where average production costs are not responsive to effort, ε is assumed to be -0.1 and -0.5. For firms with unit production costs responding to effort the absolute value of the elasticity is taken to be greater than one at -1.5 and -2. As standard TCE tends to assume technical and institutional factors are separable, we can view $\varepsilon = -0.1$ as close to standard TCE theory and hence we would expect somewhat standard predictions. On the other hand we can view $\varepsilon = -2$ as somewhat inconsistent with standard TCE and hence any conclusions that differ from those expected by standard TC analysis of the firm would indicate that analysis of interconnections between institutional and technical features of the firm do indeed matter i.e. there is intellectual value added. Two values for potential market size are used: $\alpha = 10$ and 100. It is an obvious point that the values of α used here should be interpreted ordinally. For all results $\beta=0.5$ and $\delta=1$ are used. In addition, the results report effort scaled from zero to 100 and are summarised in the Figure 3a-3h.

To interpret these diagrams recall the interpretation of the horizontal axes given above. Earlier discussion developed the idea that f'(e) is an indicator of transaction complexity. The vertical axes show the equilibrium level of organisational effort that results. First consider figure 3a. Here there is a small potential market and minimal impact of effort on production costs. The result is that we have a threshold level of f'(e) beyond which organisational effort is rational followed by a positive relationship. This result is consistent with traditional TCE as discussed above in the context of the comparative institutional method that is used. Turning to figure 3b, the

only change compared to 3a is a large potential market. Ignoring the non-monotonic relationship (for the moment) we see that increasing α shifts the relationship to the right i.e. raises the threshold f'(e) beyond which organisational effort becomes rational. This can be interpreted in the following way: a large potential market allows the management of transaction complexity over a wider range without having to internalise activity and so increase effort and hence TCs. In concrete terms this might involve greater use of subcontractors and external distributors and the like in large markets. But while the absolute impact of transaction complexity appears to interact with general market features the marginal impact appears exogenous i.e. the upward sloping parts of the diagrams have constant slope. If we adopt a monopolistic competition interpretation of the single firm model used here we can understand the shift from figure 3b to 3a (i.e. lower α) in terms of firm entry. In this case firm entry (i.e. greater competition) results in firms being more responsive to contract complexity. This endogeneity of the threshold f'(e) to α is apparent in all the diagrams i.e. when α changes with a constant ε.

The second feature of the diagrams to be discussed here concerns the impact of increasing the absolute size of ε i.e. making production costs more responsive to organisational effort. The relationship between f'(e) and e is non-monotonic, apart from when α and ε are both small in absolute terms. We can interpret this non-monotonicity as follows. Beyond the threshold f'(e) two organisational equilibria exist. The second order conditions (in the appendix) indicate that both classes of equilibria are profit maximising. The "normal" TC relationship involves increasing organisational effort, i.e. TCs, as f'(e) increases. But in addition there is a "small" organisation equilibrium; note that earlier discussion of (10a) suggests that all positive effort levels have a viable physical output. This small firm equilibrium does not display increasing TCs as transaction complexity increases. Instead there is an upper limit: there is some degree of f'(e) that renders small firm activity unviable. This upper limit depends on both α and ε. A larger potential market increases the scope for small scale activity. In addition, a larger impact of effort on unit production costs increases this scope. This result appears to identify a rationale for niche producers and the coexistence of small and large firms even with constant returns in production.

Figure 3a

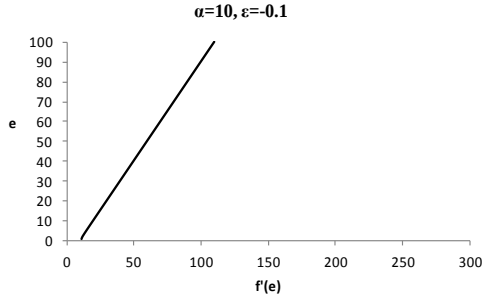


Figure 3b

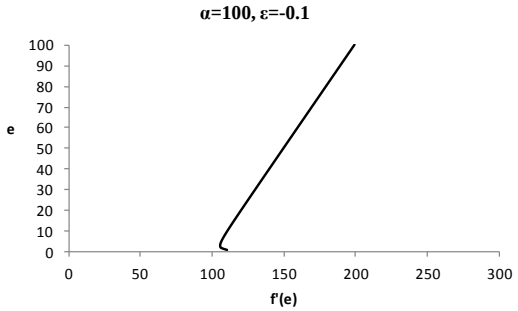


Figure 3c

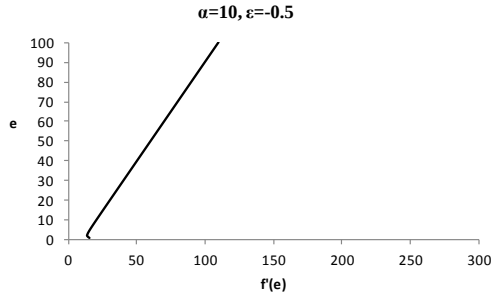


Figure 3d

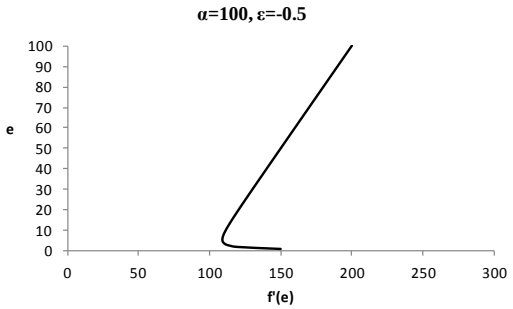


Figure 3e

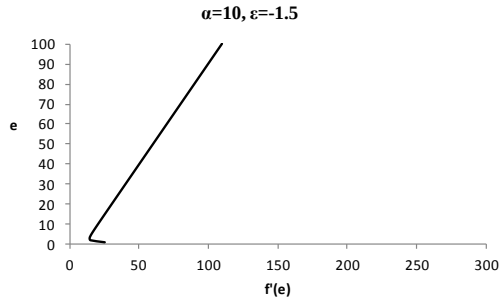


Figure 3f

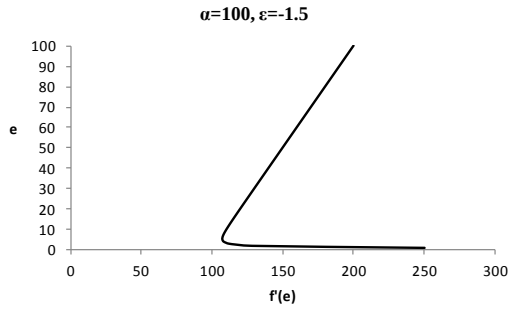


Figure 3g

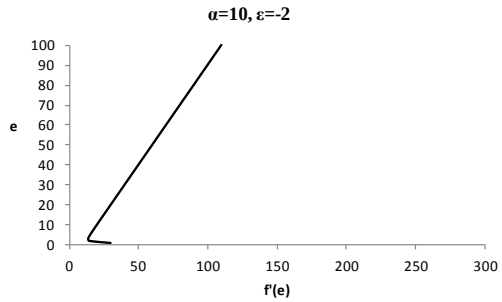
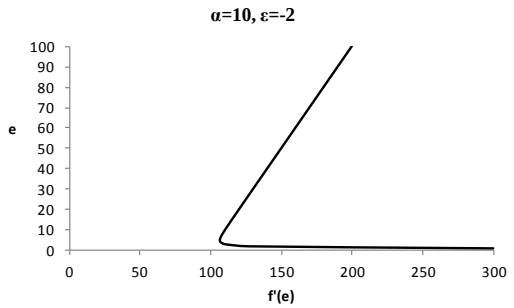


Figure 3h



To generalise this discussion of figures 3a to 3h, we see two types of firms:

1. Firms with potentially large organisational size managing increasing contracting complexity by increasing transaction costs. In the analysis presented here this effect has no upper limit but the discussion below of firm viability qualifies this. This first class of firms appear to display characteristics that are consistent with standard TCE.
2. Small firms can manage increasing contracting complexity by exploiting advantages of small organisational size, particularly when production cost advantages exist and in large markets. This effect does have an upper limit in the diagrams. This second class of firms appear to display characteristics that are not obviously consistent with standard TCE because they appear to gain their rational from interaction between technical and organisational factors.

If we view industry evolution as occurring from atomistic markets with small firms to markets with characteristically large firms, we can view this evolution in terms of some firms "jumping" from the small firm equilibrium (in 2) to a large firm equilibrium (in 1). If, as well, evolution does not reduce the absolute size of ε we can see the coexistence of small and large firms. This set of results is based on constant returns to scale i.e. it is generated by organisational factors and the impact of these factors on technical firm functioning.

<u>Firm viability</u>

Up to this point in the discussion we have ignored the important issue that any firm must earn non-negative profits for long-run viability. We will see that this qualifies, but does not fundamentally change, the analysis in the previous section. Earlier discussion defined a profit function reproduced here with a non-negativity condition:

$$\pi(x,\bar{a}(x,e),e) = \alpha x - \beta x^2 + \delta xe - x\bar{a}(x,e) - f(e) \geq 0 \qquad (13)$$

By assumption TCs (i.e. f(e)) are a linear function of effort with zero vertical intercept. Hence f(e) = ef'(e). Introducing this into (13):

$$\pi = \alpha x - \beta x^2 + \delta xe - x\bar{a}(x,e) - ef'(e) \geq 0 \qquad (13a)$$

Using (10b), reproduced here:

$$x = \frac{f'(e)}{\delta - \bar{a}_e(x,e)}$$

It follows that:

$$f'(e) = x\delta - x\bar{a}_e(x,e) \qquad (13b)$$

Substituting (13b) into (13a) and simplifying, firm viability with profit maximising output requires:

$$\alpha - \beta x - \bar{a}(x,e) + e\bar{a}_e(x,e) \geq 0 \qquad (14)$$

To compare (14) with earlier results we can use the same specific formulation: β=0.5 and $\bar{a} = x^0 e^{\varepsilon}$. Using this formulation and re-arranging (14) implies a constraint on output:

$$x \leq 2[\alpha + (\varepsilon - 1)e^{\varepsilon}] \qquad (14a)$$

Using (10b), to ensure profit maxising effort, this output constraint for viability can be re-written as a constraint on f'(e):

$$f'(e) \leq 2[\alpha - \alpha\varepsilon e^{(\varepsilon-1)} + (\varepsilon-1)e^{\varepsilon} - \varepsilon(\varepsilon-1)e^{(2\varepsilon-1)}] \qquad (15)$$

The inequality (15) indicates that firm viability, with firms profit maximising with respect to both output and effort, is subject to an upper constraint on transaction complexity i.e. excessive transaction complexity produces negative profitability. To compare the implications of this inequality with earlier results we can use the particular parameter values for figures 3a and 3h with the intermediate cases being somewhat obvious in the light of earlier discussion. Figures 4a and 4b show the results of this exercise: 4a uses α=10 and ε=-0.1, and 4b uses α=100 and ε=-2. Values of f'(e) to the left of these curves are viable equilibria.

Figure 4a

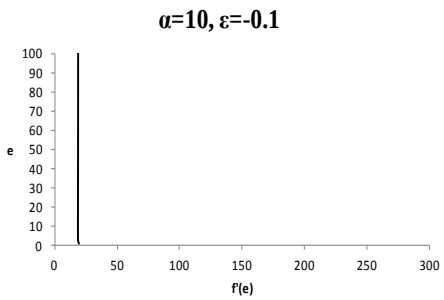
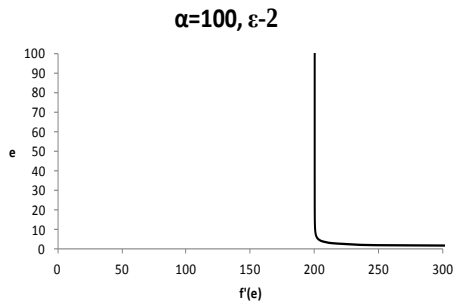


Figure 4b



In figure 4a the constraint produces an upper limit on organisational size (e) i.e. only the bottom segment of the equilibria identified in figure 3a is viable. While only small firms are viable, given the parameter values used in 4a, this does not change the basic relevance of the conclusions drawn about TCE i.e. increasing transaction complexity increases TCs but subject to an upper limit. Figure 4b should be compared with the

equilibria identified in figure 3h. Examination of these two diagrams indicates that all the small firm equilibria are viable but viable large firms have a maximum possible level of effort. Once again, the conclusions drawn are qualified but not subject to fundamental change when viability issues are introduced.

Continuing the themes set out above we can reiterate three issues. First the viability constraints identified in figures 4a and 4b are based on constant returns in production. Hence the profitability constraint is produced by organisational factors and their interaction with technical issues, not the technical production issues directly. Secondly, we can interpret the constraint on large firm organisational size, but not the small firm solutions, in terms of a standard analysis of control loss. Hence even though a constant elasticity of effort on unit production costs is used here the interactions of the model produce a somewhat standard result. Finally, if we interpret the change in α in figure 4b compared to 4a in terms of firm entry (and monopolistic competition) we can see how a long-run equilibrium constraint becomes binding and controls firm size because of transaction complexity constraining organisational size.

Conclusion

The discussion presented above has been motivated by three key themes, as set out in the introduction: to integrate TCs into a standard model of the firm; to examine the interaction between organisational factors (TCs) and standard demand-cost factors; and to analyse key propositions of transaction cost economics (TCE) with the general model. Given these themes, two sets of conclusions can be reinforced in this conclusion. First, when analysis is based on significant interaction between organisational effort and production costs two possible organisational equilibria can exist. First we have a “normal” relationship that beyond some threshold increasing transaction complexity results in increasing organisational effort; this possibility was denoted as a large firm equilibrium. But in addition a small firm equilibrium exists in which increasing transaction complexity reduces organisational size, but with an upper limit. This result of (potential) multiple organisational equilibria is consistent with results from some ago by Pagano (1992) and Pagano and Rowthorn (1994). While both these latter papers and the approach developed here have grown out of transaction cost theory and have similar conclusions, the theoretical frameworks used are somewhat different. But collectively they can be seen as suggesting an important

organisational feature that is often overlooked i.e. the possibility of non-unique solutions.

The second key conclusion concerns firm viability. When a non-negative profit constraint is introduced this qualifies but does not fundamentally change the findings. All the small firm solutions are viable but the large firm solutions are subject to an upper bound. This finding was interpreted as the equivalent of a “control loss” constraint that exists even if average production costs have a homothetic relationship with organisational effort i.e. there is a constant elasticity. In short these findings collectively indicate that analysis of the interaction between organisational and technical aspects of the firm using a formal method does indeed add value in terms of our understanding.

The second matter that can be briefly considered in this conclusion is the single firm framework upon which the analysis is based. In the text this was interpreted either as a monopoly model or in terms of monopolistic competition. In the latter context, the analysis based on constant returns in production is revealing. The interaction of firm entry and organisational factors generates a movement to a zero profit long-run equilibrium i.e. when the viability constraint binds. Reduced market size increases the impact of transaction complexity on organisational size. This reinforces the analytical logic that starts with the impact of organisational conditions on organisational effort and then in turn effort determines physical output. This is consistent with the general approach to the firm that organisation matters and is the key driver in firm development. But an obvious possible development here, beyond the scope of the existing discussion, is to base analysis on other market structures. An obvious possibility here is to use a Cournot-type oligopoly context. Using a simple completely linear model Dietrich and Krafft (2012) undertake such an exercise and shows that the existence of firms depends on market power as well as efficiency. But this latter discussion lacks the rich analysis resulting from the non-linear cost structure used here. Hence incorporating the framework developed here into an oligopoly context offers a fruitful area for future work.

Appendix

This appendix presents second order conditions for the key results presented in the text.

The initial non-TC model had a first order condition for maximum profit:

$$\frac{\partial \pi}{\partial x} = \alpha - 2\beta x - a(x) - xa'(x) = 0 \qquad \text{[A1a]}$$

Here a'(x) indicates the nature of production based scale economies. To complete this simple introductory model we have the second order condition:

$$\frac{\partial^2 \pi}{\partial x^2} = -2\beta - 2a'(x) - xa''(x) < 0 \qquad \text{[A1b]}$$

The model that includes organisational effort has first order conditions for maximum profit given by:

$$\frac{\partial \pi}{\partial x} = \alpha - 2\beta x + \delta e - \bar{a}(x,e) - x\bar{a}_x(x,e) = 0 \qquad \text{[A2a]}$$

$$\frac{\partial \pi}{\partial e} = \delta x - x\bar{a}_e(x,e) - f'(e) = 0 \qquad \text{[A2b]}$$

The second order condition, based on the Hessian being negative definite, requires:

$$-2\beta - 2\bar{a}_x(x,e) - x\bar{a}_{xx}(x,e) < 0 \qquad \text{[A3a]}$$

$$-x\bar{a}_{ee}(x,e) - f''(e) < 0 \qquad \text{[A3b]}$$

$$[-2\beta - 2\bar{a}_x(x,e) - x\bar{a}_{xx}(x,e)][-x\bar{a}_{ee}(x,e) - f''(e)] - [\delta - \bar{a}_e(x,e) - x\bar{a}_{ex}(x,e)]^2 > 0 \qquad [A3c]$$

We finally show that the two classes of solutions ("normal" and "small" organisations) identified in figures 3a-3f are both consistent with the second order conditions [A3a, b, c]. The figures in the text are based on a constant elasticity unit production cost function with constant returns in production:

$$\bar{a} = x^0 e^{\varepsilon} \qquad \text{[A4a]}$$

Hence:

$$\bar{a}_x(x,e) = \bar{a}_{xx}(x,e) = 0 \qquad \text{[A4b]}$$

$$\bar{a}_e(x,e) = \varepsilon e^{(\varepsilon-1)} \qquad \text{[A4c]}$$

$$\bar{a}_{ee}(x,e) = \varepsilon(\varepsilon-1)e^{(\varepsilon-2)} \qquad \text{[A4d]}$$

$$\bar{a}_{ex} = 0 \qquad \text{[A4e]}$$

In addition, from figure 1 it follows that:

$$f''(e) = 0 \qquad \text{[A4f]}$$

Finally figures 3a-3f are based on specific parameter values:

$$\beta=0.5 \text{ and } \delta=1 \qquad \text{[A4g]}$$

Using [A4a]-[A4g] and [A3a]

$$-1 - 2(0) - x(0) < 0$$

Using [A4a]-[A4g] and [A3b]

$$-x\varepsilon(\varepsilon - 1)e^{(\varepsilon-2)} - 0 < 0$$

All viable x and e are positive, and as ε is negative (by assumption) it follows that this part of the second order condition is negative.

Using [A4a]-[A4g] and [A3c]

$$[-1][-x\varepsilon(\varepsilon - 1)e^{(\varepsilon-2)}] + [1 - \varepsilon e(\varepsilon - 1)][1 - \varepsilon e(\varepsilon - 1)] > 0$$

All viable x and e are positive, and ε is negative by assumption, it follows that this part of the second order condition is positive.

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